PEARSON

MATHEMATICAL METHODS

QUEENSLAND

EXAM PREPARATION WORKBOOK

12

UNITS 3 & 4

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MATHEMATICAL METHODS 12

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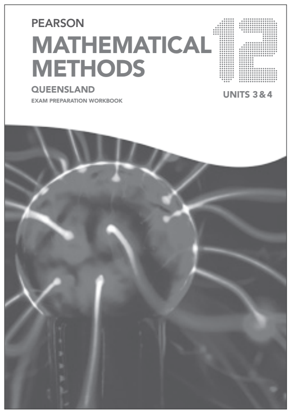

About this **Pearson Mathematical Methods 12 Exam Preparation Workbook**

The purpose of the **Pearson Exam Preparation Workbook** is to assist students in their preparation for the QCAA external exams. Answering previous external exam questions is an effective way to do this, as it offers well-constructed questions at the appropriate level.

This **Pearson Exam Preparation Workbook** includes previous external exam questions from Victoria. Given that both the syllabuses and the access to allowed technologies varies across the states, the author has reviewed questions from years 2000 to 2017 to select those questions that align with the QCAA syllabus.

These questions were then categorised using the QCAA three levels of difficulty: simple familiar, complex familiar and complex unfamiliar. This matches the QCAA external exam structure, which indicates that in Paper 1 and Paper 2, marks will be allocated in the following approximate proportions:

- 60% simple familiar
- 20% complex familiar
- 20% complex unfamiliar.

The source of each question in the **Pearson Exam Preparation Workbook** is referenced at the start of the question. At times, there may be some variation between the notation used in the questions and that used by the QCAA; the authors have made note of this within the worked solution as applicable.

Each worked solution has indicative mark allocations. As official marking schemes are not released by state examining bodies, the mark allocations in the **Pearson Exam Preparation Workbook** are based on the author's and reviewer's on-balance judgement and their teaching experience.

Writing and development team

We are grateful to the following people for their time and expertise in contributing to **Pearson Mathematical Methods 12 Exam Preparation Workbook**.

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How to use this workbook

Pearson Mathematical Methods 12 Queensland Exam Preparation Workbook, Units 3 & 4

This exam preparation workbook has been developed to assist students in their preparation for the QCAA external exams. It provides previous external exam questions from Victoria that align with the QCAA syllabus. All questions are categorised into the QCAA three levels of difficulty—simple familiar, complex familiar and complex unfamiliar—to match the QCAA external exam structure.

The questions have been grouped into convenient sets, with the intention that each question set is tackled in one sitting.

Levels of difficulty

Levels of difficulty are indicated using a three-striped label.

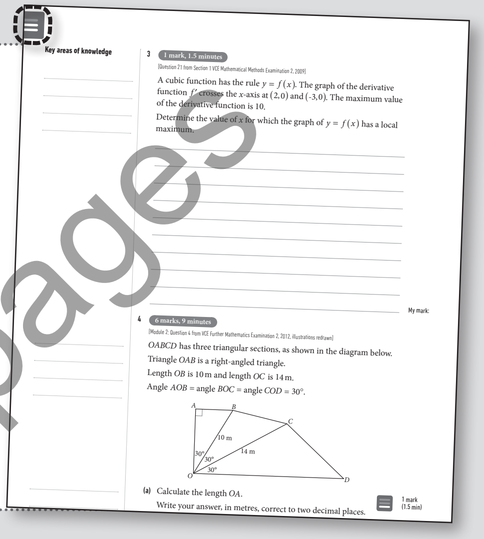
Simple familiar: Complex familiar:

Complex unfamiliar:

These are used in two ways:

1 To show the level of difficulty for the whole question set

2 To show the level of difficulty of individual question parts when they differ from that of the question set level. In such cases, all parts of that question are labelled.



Key areas of knowledge

3 1 mark, 1.5 minutes

[Question 21 from Section 1 VCE Mathematical Methods Examination 2, 2009]

A cubic function has the rule $y = f(x)$. The graph of the derivative function f' crosses the x-axis at $(2,0)$ and $(-3,0)$. The maximum value of the derivative function is 10.

Determine the value of x for which the graph of $y = f(x)$ has a local maximum.

My mark:

4 6 marks, 9 minutes

[Module 2: Question 4 from VCE Further Mathematics Examination 2, 2012, illustrations redrawn]

$OABCD$ has three triangular sections, as shown in the diagram below.

Triangle OAB is a right-angled triangle.

Length OB is 10 m and length OC is 14 m.

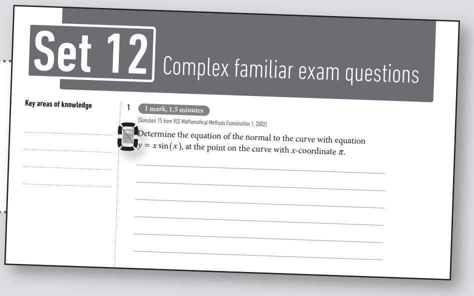
Angle AOB = angle BOC = angle COD = 30°.

(a) Calculate the length OA.

Write your answer, in metres, correct to two decimal places. 1 mark (1.5 min)

Technology-free questions

Questions to be completed without the use of any technology will have a crossed-out calculator image beside them:



Set 12 Complex familiar exam questions

Key areas of knowledge

1 1 mark, 1.5 minutes

[Question 15 from VCE Mathematical Methods Examination 1, 2002]

Determine the equation of the normal to the curve with equation $y = x\sin(x)$, at the point on the curve with x-coordinate π.

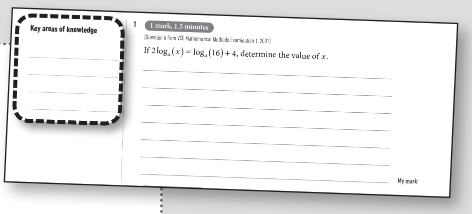
Get yourself exam ready using this 5-step preparation sequence

Step 1: Key areas of knowledge

The purpose of making these notes is to first identify ***what*** is required to be done, and ***how*** it might be done, ***without*** doing it at this stage.

For each question, note the topic(s) of mathematics it draws on, formulas you think will be needed, and any other comments you feel will help you work out the answer.

Then move on to the next question in that set.



Key areas of knowledge

1 1 mark, 1.5 minutes

[Question 6 from VCE Mathematical Methods Examination 1, 2001]

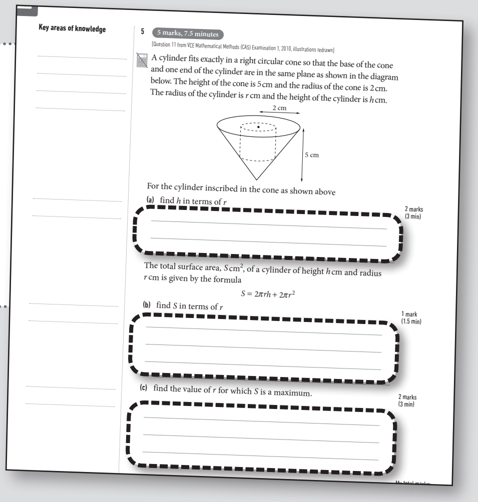
If $2\log_a(x) = \log_a(16) + 4$, determine the value of x.

My mark:

Step 2: Complete questions

Complete all the questions within the question set using the space provided.

Question sets have been structured according to level of difficulty, with each question set covering a range of topics from the syllabus.


Key areas of knowledge

5 5 marks, 7.5 minutes

A cylinder fits exactly in a right circular cone so that the base of the cone and one end of the cylinder are in the same plane as shown in the diagram below. The height of the cone is 5 cm and the radius of the cone is 2 cm. The radius of the cylinder is r cm and the height of the cylinder is h cm.

For the cylinder inscribed in the cone as shown above

(a) find h in terms of r — 2 marks (3 min)

The total surface area, S cm², of a cylinder of height h cm and radius r cm is given by the formula

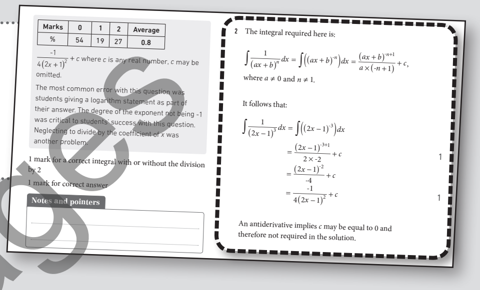
$$S = 2\pi rh + 2\pi r^2$$

(b) find S in terms of r — 1 mark (1.5 min)

(c) find the value of r for which S is a maximum. — 2 marks (3 min)

Step 3: Check your answer

Review and mark your answers according to the solutions provided in the corresponding worked solutions.



Marks	0	1	2	Average
%	54	19	27	0.8

$\frac{-1}{4(2x+1)^2} + c$ where c is any real number, c may be omitted.

The most common error with this question was students giving a logarithm statement as part of their answer. The degree of the exponent not being −1 was critical to students' success with this question. Neglecting to divide by the coefficient of x was another problem.

1 mark for a correct integral with or without the division by 2

1 mark for correct answer

Notes and pointers

2 The integral required here is:

$$\int \frac{1}{(ax+b)^n}dx = \int \left((ax+b)^{-n}\right)dx = \frac{(ax+b)^{-n+1}}{a \times (-n+1)} + c,$$

where $a \neq 0$ and $n \neq 1$.

It follows that:

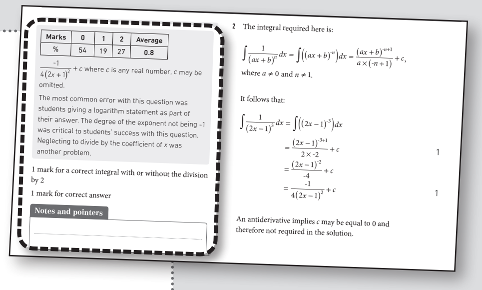
$$\int \frac{1}{(2x-1)^3}dx = \int \left((2x-1)^{-3}\right)dx = \frac{(2x-1)^{-3+1}}{2 \times -2} + c = \frac{(2x-1)^{-2}}{-4} + c = \frac{-1}{4(2x-1)^2} + c$$

An antiderivative implies c may be equal to 0 and therefore not required in the solution.

Step 4: Examination report and reflection

Review the marks obtained from past students, read the information in the **Examination report** section (where available) and reflect on your own solution.

Use the **Notes and pointers** section to write down any relevant key reminders to yourself about common errors, key rules etc.

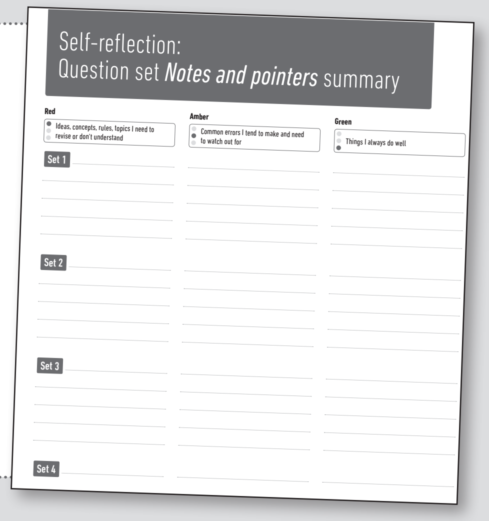


Step 5: Self-reflection: Question set notes and pointers summary

Reflect on all the questions within one set, review your comments in the individual Notes and pointers sections, and use these to complete a summary of the overall question set.

Use the **Red**, **Amber** and **Green** categories to note what you need to revise or don't understand, what you need to watch out for, and what you did well.

Once all sets are completed, these summaries will help in giving you direction on where to focus your further revision.


Self-reflection: Question set *Notes and pointers* summary

Red — Ideas, concepts, rules, topics I need to revise or don't understand

Amber — Common errors I tend to make and need to watch out for

Green — Things I always do well

Set 1

Set 2

Set 3

Set 4

Contents

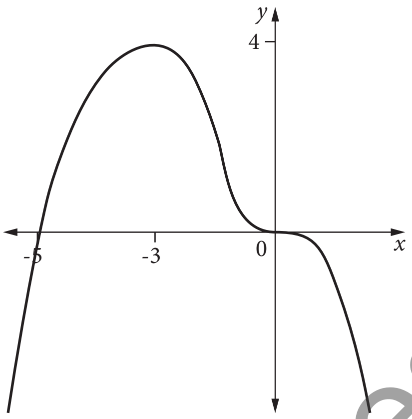
Set 2 Simple familiar exam questions

Key areas of knowledge

1 1 mark, 1.5 minutes

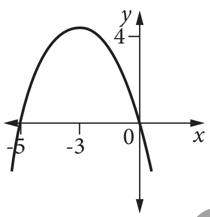
[Question 9 from Section 1 VCE Mathematical Methods (CAS) Examination 2, 2011, illustrations redrawn]

The graph of the function $y = f(x)$ is shown below.

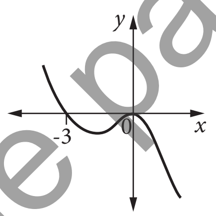


Which of the following could be the graph of the derivative function $y = f'(x)$?

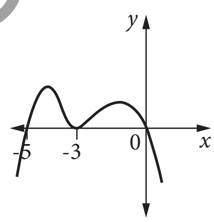
A



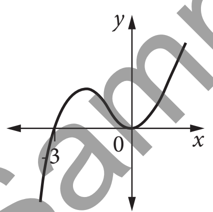
B



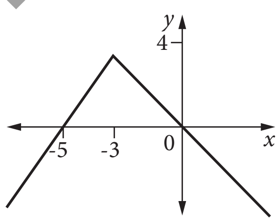
C



D



E



My mark:

2 4 marks, 6 minutes

[Question 5 from VCE Mathematical Methods Examination 1, 2013]

(a) Solve the equation $2\log_3(5) - \log_3(2) + \log_3(x) = 2$ for x. 2 marks (3 min)

(b) Solve the equation $3^{-4x} = 9^{6-x}$ for x. 2 marks (3 min)

My total marks:

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Key areas of knowledge

3 3 marks, 4.5 minutes

[Question 2 from VCE Mathematical Methods (CAS) Examination 1, 2015]

Let $f'(x) = 1 - \frac{3}{x}$, where $x > 0$.

Given that $f(e) = -2$, find $f(x)$.

My mark:

4 5 marks, 7.5 minutes

[Question 5 from VCE Mathematical Methods (CAS) Examination 1, 2014]

Consider the function $f(x) = 3x^2 - x^3$, where $x \in [-1, 3]$.

(a) Find the coordinates of the stationary points of the function. 2 marks (3 min)

(b) Find the area enclosed by the graph of the function and the horizontal line given by $y = 4$. 3 marks (4.5 min)

My total marks:

Key areas of knowledge

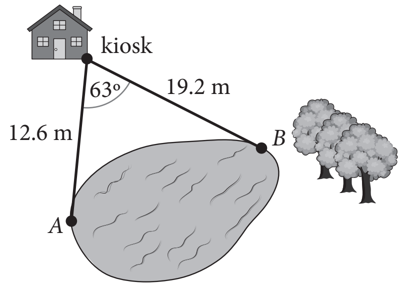
5 1 mark, 1.5 minutes

[Module 2: Question 2 from Section B VCE Further Mathematics Examination 1, 2013, illustrations redrawn]

The distances from a kiosk to points A and B on opposite sides of a pond are found to be 12.6 m and 19.2 m respectively.

The angle between the lines joining these points to the kiosk is 63°.

The distance, in m, across the pond between points A and B can be found by evaluating



A $\frac{1}{2} \times 12.6 \times 19.2 \times \sin(63°)$

B $\frac{19.2 \times \sin(63°)}{12.6}$

C $\sqrt{12.6^2 + 19.2^2}$

D $\sqrt{12.6^2 + 19.2^2 - 2 \times 12.6 \times 19.2 \times \cos(63°)}$

E $\sqrt{s(s-12.6)(s-19.2)(s-63)}$, where $S = \frac{1}{2}(12.6 + 10.2 + 63)$

My mark:

6 5 marks, 7.5 minutes

[Question 4 from VCE Mathematical Methods Examination 1, 2008]

The function

$$f(x) = \begin{cases} k\sin(\pi x) & \text{if } x \in [0,1] \\ 0 & \text{otherwise} \end{cases}$$

is a probability density function for the continuous random variable X.

(a) Show that $k = \frac{\pi}{2}$.

2 marks (3 min)

(b) Find $P\left(X \le \frac{1}{4} \mid X \le \frac{1}{2}\right)$.

3 marks (4.5 min)

My total marks:

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Simple familiar worked solutions and examination report

Set 2

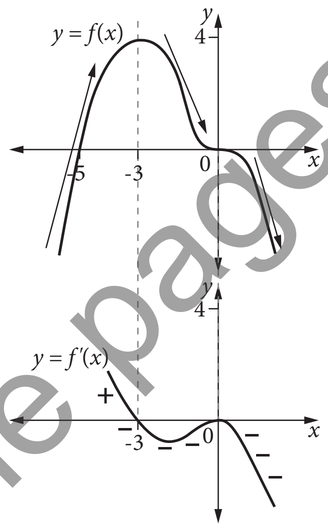
Examination report comments | **Worked solutions** | **Marks**

% A	% B	% C	% D	% E	% No Answer
7	77	6	7	2	0

Correct option is **B**.

Notes and pointers

1 In the following diagram, observing the behaviour of the graph from left to right, it can be seen that the value of the gradient of a tangent to the curve at any point changes. For $x < -3$ the gradient changes from positive to a turning point (zero) at $x = -3$, then negative between $-3 < x < 0$ to a stationary point of inflection (zero) again at $x = 0$, and finally to an increasingly negative gradient for $x > 0$. 1



Marks	0	1	2	Average
%	13	32	54	1.4

Many students solved this equation correctly.
A disappointing number of students could not combine all parts of the logarithms into a single expression.

1 mark for a correct single logarithm
1 mark for correct answer

Notes and pointers

2 (a) For the expression $\log_3(x)$ to exist, $x > 0$:

$$2\log_3(5) - \log_3(2) + \log_3(x) = 2$$
$$\log_3(5^2) - \log_3(2) + \log_3(x) = 2$$
$$\log_3\left(\frac{5^2}{2}\right) + \log_3(x) = 2$$
$$\log_3\left(\frac{25x}{2}\right) = 2 \quad 1$$
$$\frac{25x}{2} = 3^2$$
$$25x = 18$$
$$x = \frac{18}{25} \quad 1$$

Given that $x > 0$, the solution is reasonable.

Examination report comments | **Worked solutions** | **Marks**

Marks	0	1	2	Average
%	14	23	64	1.5

The majority of incorrect responses involved $9 = 3^3$.

1 mark for correct exponential equation with the same base

1 mark for the correct answer

Notes and pointers

(b) Find a common exponential base for both sides of the equation. In this case it is 3.

$3^{-4x} = 9^{6-x}$

$3^{-4x} = \left(3^2\right)^{6-x}$

$3^{-4x} = 3^{12-2x}$ — 1

Given that the bases are equal, equate the exponents:

$-4x = 12 - 2x$

$-2x = 12$

$x = -6$ — 1

Marks	0	1	2	3	Average
%	14	14	21	51	2.1

Most students identified that the antiderivative involved a logarithmic expression. Evaluation of the constant of antidifferentiation caused some difficulties.

1 mark for a correct integral with or without the constant of integration

1 mark for $c = 1 - e$

1 mark for the correct answer

Notes and pointers

3 $f(x) = \int\left(1 - \frac{3}{x}\right)dx$

$= x - 3\log_e(x) + c, x > 0$ — 1

$f(e) = -2$

$-2 = e - 3\log_e(e) + c$

$-2 = e - 3 + c$

$c = -2 + 3 - e$

$c = 1 - e$ — 1

Therefore: $f(x) = x - 3\log_e(x) + 1 - e,\ x > 0$ — 1

Marks	0	1	2	Average
%	14	20	66	1.5

Students must be vigilant in ensuring they answer the specific question. In this question, coordinates were required and not simply x-values. Some students omitted the turning point $(0,0)$ and others incorrectly found the x-intercepts.

1 mark for correct x-values

1 mark for correct coordinates

Notes and pointers

4 (a) $f'(x) = 6x - 3x^2$ and for stationary points $f'(x) = 0$.

Hence:

$f'(x) = 3x(2 - x)$

$3x(2 - x) = 0$

$x = 0$ or $x = 2$ — 1

At $x = 0$,

$f(0) = 3(0)^2 - (0)^3$

$= 0$

At $x = 2$,

$f(2) = 3(2)^2 - (2)^3$

$= 12 - 8$

$= 4$

The coordinates of the stationary points are $(0,0)$ and $(2,4)$. — 1

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Examination report comments

Marks	0	1	2	3	Average
%	25	27	27	21	1.5

Method 1

$$\text{Area} = 12 - \int_{-1}^{2}\left(3x^2 - x^3\right)dx$$
$$= 12 - \left[x^3 - \frac{x^4}{4}\right]_{-1}^{2}$$
$$= 12 - \left[(8-4) - \left(-1 - \frac{1}{4}\right)\right]$$
$$= 6\frac{3}{4}\left(\text{or } \frac{27}{4} \text{ or } 6.75\right)$$

Method 2

$$\int_{-1}^{2}\left(4 - \left(3x^2 - x^3\right)\right)dx = \left[4x - x^3 + \frac{x^4}{4}\right]_{-1}^{2}$$
$$= (8 - 8 + 4) - \left(-4 + 1 + \frac{1}{4}\right) = 6\frac{3}{4}$$

Most students knew to seek a difference of two areas and were adept with basic integration; however, quite often arithmetic errors in evaluations or the incorrect use of negative signs marred their progress towards acquiring full marks. Some students unnecessarily 'overworked' the problem by creating three or four integrations, increasing the likelihood of an error. A few students took a more direct route that involved symmetry of the curve.

1 mark for a clear indication that the enclosed area is between $x = -1$ and $x = 2$

1 mark for correct integral

1 mark for correct answer

Notes and pointers

Worked solutions **Marks**

(b) Determine the coordinates of the points of intersection:

Given that at $x = 2,\ y = 4$:

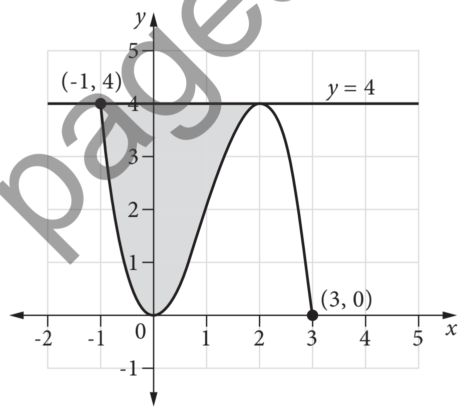
$$4 = 3x^2 - x^3$$
$$x^3 - 3x^2 + 4 = 0$$
$$(x-2)\left(x^2 + kx - 2\right) = 0$$

Equate coefficients:

$$-2x^2 + kx^2 = -3x^2$$
$$(k-2)x^2 = -3x^2$$
$$k - 2 = -3$$
$$k = -1$$

Therefore:

$$(x-2)\left(x^2 - x - 2\right) = 0$$
$$(x-2)(x-2)(x+1) = 0$$
$$x = 2 \text{ or } x = -1$$



1

The area between the horizontal line $y = 4$ and the function $f(x) = 3x^2 - x^3$ for $-1 \le x \le 3$ is shaded in the diagram above. The integral to find this area as shown in the Examiners report is:

$$\int_{-1}^{2}\left(4 - \left(3x^2 - x^3\right)\right)dx = 6\frac{3}{4}$$

2

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Examination report comments

% A	% B	% C	% D	% E	% No Answer
5	8	4	81	2	0

Correct option is **D**.

Notes and pointers

Worked solutions | **Marks**

5 Given that two side lengths and the included angle are given, the cosine rule can be applied to determine the distance AB, opposite the angle 63°.

Using $c^2 = a^2 + b^2 - 2ab\cos(C)$, with $a = 12.6$ and $b = 19.2$.

$$AB = \sqrt{12.6^2 + 19.2^2 - 2 \times 12.6 \times 19.2 \times \cos(63°)}$$ 1

Marks	0	1	2	Average
%	38	16	46	1.2

Although many students answered this question satisfactorily, there was a significant number of vanishing negative signs, incorrect indefinite integrals, incorrect substitutions or incorrect algebraic manipulation. It was pleasing to see that only a few students left off the '*dx*'.

1 mark for a correct integral equated to 1

1 mark for correct answer

Notes and pointers

6 **(a)**

$$\int_0^1 k\sin(\pi x)\,dx = 1$$

$$\left[-\frac{k}{\pi}\cos(\pi x)\right]_0^1 = 1$$ 1

$$-\frac{k}{\pi}\cos(\pi) + \frac{k}{\pi}\cos(0) = 1$$

$$2\frac{k}{\pi} = 1$$

$$k = \frac{\pi}{2}$$ 1

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Examination report comments

Marks	0	1	2	3	Average
%	42	18	16	24	1.3

$$P\left(X \le \frac{1}{4} \mid X \le \frac{1}{2}\right) = \frac{P\left(x \le \frac{1}{4}\right)}{P\left(x \le \frac{1}{2}\right)}$$

$$\frac{\int_0^{\frac{1}{4}} k\sin(\pi x)\,dx}{\int_0^{\frac{1}{2}} k\sin(\pi x)\,dx} = \frac{\int_0^{\frac{1}{4}} k\sin(\pi x)\,dx}{0.5} = \frac{2-\sqrt{2}}{2}$$

Few students used symmetry of the probability density function to obtain the denominator. However, it appeared that many students found this question both difficult and complicated. Many recognised the need for conditional probability but thought the required intersection was between $\frac{1}{4}$ and $\frac{1}{2}$ rather than between 0 and $\frac{1}{4}$. Some students correctly navigated complicated expressions and their evaluation without recognising how much simply cancelled (for example, k), while others, whose method was correct, were let down by poor manipulation skills and were left with π in the answer.

1 mark for recognising or calculating $P\left(x \le \frac{1}{2}\right) = 0.5$

1 mark for calculating $P\left(x \le \frac{1}{4}\right)$

1 mark for correct answer

Notes and pointers

Worked solutions **Marks**

(b) This is a conditional probability question where:

$$P\left(X \le \frac{1}{4} \mid X \le \frac{1}{2}\right) = \frac{P\left(x \le \frac{1}{4}\right)}{P\left(x \le \frac{1}{2}\right)},$$

and by symmetry of the sine function with a period of 2, over the domain [0, 1]:

$$P\left(x \le \frac{1}{2}\right) = 0.5$$ 1

$$P\left(x \le \frac{1}{4}\right) = \int_0^{0.25}\left(\frac{\pi}{2}\sin(\pi x)\right)dx$$

$$= \frac{\pi}{2}\left[\frac{-\cos(\pi x)}{\pi}\right]_0^{0.25}$$

$$= \frac{-1}{2}\left[\cos(0.25\pi) - \cos(0)\right]$$

$$= \frac{-1}{2}\left[\frac{1}{\sqrt{2}} - 1\right]$$ 1

Hence:

$$P\left(X \le \frac{1}{4} \mid X \le \frac{1}{2}\right) = \frac{\frac{-1}{2}\left[\frac{1}{\sqrt{2}} - 1\right]}{0.5}$$

$$= 1 - \frac{1}{\sqrt{2}}$$

$$= \frac{2-\sqrt{2}}{2}$$ 1

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